



How to select a suitable ice class – a discussion on goal- and risk-based design of polar ships

Opinion paper

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Abstract. Global warming has led to significant changes in the Arctic ice environment during the past decades, resulting in high expectations for future polar shipping. Ships going through ice-covered water need to strengthen their structure to resist ice loading. Nonetheless, over-strengthening can result in excessive lightweight and hamper the profitability of the operation. How to select the most suitable ice class is usually a primary question of ship owners, and is also the starting point of polar ship design. The current rule-based design methodology of polar ships has several limitations that may restrain the feasibility of more efficient and flexible designs. Goal- and risk-based design has emerged as an alternative due to its bespoke features and better adaptability to the mission of ships. Moreover, goal- and risk-based design can incorporate limit state design methodology, which can effectively integrate the random nature of ice loads with the extent of structure damage. To date, some gaps exist before the actual use of this new approach. This paper aims to present an overview of goal- and risk-based design approach in comparison to rule-based design, and make a discussion on the way ahead to turn this from concept to practice.

Keywords: icegoing ships, icebreaking, polar ships, POLARIS, Arctic.

1. Limitations of rule-based design

The International Maritime Organization (IMO) introduced the International Code for Ships Operating in Polar Waters (Polar Code) to enhance safe navigation in polar waters (IMO, 2015). This code came into force in 2017 and will soon be effective for 10 years. In this code, IMO provides the Polar Operational Limit Assessment Risk Indexing System (POLARIS) to be used as guidance to select the ice class of newbuilt ships. POLARIS is supposed to be used together with the Unified Requirements (UR) for Polar Ships developed by the International Association of Classification Societies (IACS, 2016). A risk index can be calculated based on ice conditions of the operation area, together with the polar class of a ship. The risk index indicates different levels of operational risks, and corresponds to different operational measures that shall be taken, including

normal operation, elevated operational risk, and operation subject to special consideration. In practice, a ship owner can use POLARIS to evaluate options of using different polar classes to operate in its designated area, and select the most suitable one based on the risk assessment results.

As discussed by Bergström et al. (2020), POLARIS is essentially goal-based, which allows the selection of the polar class according to its mission. Nevertheless, the goal here only involves areas of operation, but does not cover some other important aspect, such as the operation mode. This can be demonstrated by the following examples.

Example 1: Merchant vessel, ship A, and polar research vessel, ship B, operate in the same polar region. According to IMO POLARIS and IACS UR, the two ships should have the same ice class. However, ship B maneuvers much more frequently in ice to carry out

scientific activities, while ship A prioritizes avoidance of ice to reduce energy consumption. Compared to Ship A, Ship B should have a higher level of strengthening, especially in the stern area, due to its extensive exposure to ice.

Example 2: Merchant vessel, ship C, has an open water bow. She operates independently only in the ice floe area and is escorted by icebreakers when encountering large-scale ice. According to IACS UR, the design ice load is calculated assuming a glancing impact scenario, while in fact the ship may never encounter such scenarios.

Example 3: A passenger vessel, ship D, an oil tanker, ship E, and a bulk carrier, ship F, operate in the same area. According to IACS UR and IMO POLARIS, the three ships shall have the same level of strengthening. However, the damage of these ships may incur rather different consequences for humans, the economy, and the environment.

These limitations may result in a homogenized design, which is not optimized in terms of efficiency and profitability. Goal- and risk-based design can provide improvement to the POLARIS – IACS UR methodology by enabling more flexible and bespoke design.

2. Development of goal- and risk- based design concept in polar ship design

The essential idea of goal-based design of polar ships is to determine the design ice loads by integrated consideration of the mission of the ship, area of operation, mode of operation, and consequences of damage. It is often coupled with risk-based design, which makes decisions based on the assessment of risk in a probabilistic manner. Risk is typically calculated as the probability of damage multiplied by the extent of consequence. The whole process then starts with defining the goal and ends with the assessment of risk, based on which the scantlings can be determined.

With goal- and risk-based design, the design ice loads of different structure members at different locations of the ship can be calculated independently, therefore designed according to different ice classes, although on the same ship. In comparison, in current ice class rules, an area factor (see Table 1) is assigned to different locations of a ship – the relative magnitude of ice loads between different locations (e.g., at the bow and stern shoulder) never changes.

Table 1. Hull area factors specified by IACS UR (IACS, 2016).

Hull area		Polar Class						
		PC1	PC2	PC3	PC4	PC5	PC6	PC7
Bow (B)	All	1.00	1.00	1.00	1.00	1.00	1.00	1.00
Bow Intermediate (BI)	Icebelt	0.9	0.85	0.85	0.8	0.8	1.00*	1.00*
	Lower	0.7	0.65	0.65	0.6	0.55	0.55	0.5
	Bottom	0.55	0.5	0.45	0.4	0.35	0.3	0.25
Midbody (M)	Icebelt	0.7	0.65	0.55	0.55	0.5	0.45	0.45
	Lower	0.5	0.45	0.4	0.35	0.3	0.25	0.25
	Bottom	0.3	0.3	0.25	**	**	**	**
Stern (S)	Icebelt	0.75	0.7	0.65	0.6	0.5	0.4	0.35
	Lower	0.45	0.4	0.35	0.3	0.25	0.25	0.25
	Bottom	0.35	0.3	0.3	0.25	0.15	**	**

Lloyd’s Register (LR) introduced the icebreaker(+) notation in 2022, aiming to facilitate bespoke design of polar ships as an alternative to prescriptive design (Lloyd’s Register, 2022). Designers and operators are allowed to define operational scenarios and calculate bespoke area factors based on the scenario. This is a major step towards goal- and risk-based design, which in principle enables alternative design.

Major progress has been achieved on the development of goal- and risk-based design of polar ships over the past ten years. Bergström et al. (2016) made a comprehensive assessment of the applicability of goal- and risk-based design on Arctic maritime transport systems. This includes the architecture of an Arctic fleet as well as the design of individual ships. Ehlers et al. (2017) proposed a mission-based approach for the structure design of Arctic ships and offshore

structures, which is, in principle, equivalent to goal-based design. The core of this approach is a probabilistic method capable of predicting ice loads in the long term, facilitating differentiated ice loads depending on the goal of a ship. The return period of ice loads is linked to the limit state of ship structures. Kujala et al. (2019) made a review of risk-based design of polar ships, focusing on available methods to support risk-based design. Later, Bergström et al. (2022) proposed a goal-based approach to select ships' ice class based on an improved probabilistic ice load prediction method, capable of accounting for more detailed ice information.

It can be concluded from the research so far that some consensus has been reached on the procedure of goal- and risk-based design. Figure 1 shows a flowchart of the main elements of goal- and risk-based design, including defining the goal, probabilistic modeling of ice loads, and modeling of damage and consequence. Next, we will go through these elements sequentially.

1. **Defining the goal.** This is to convert the ship owners' needs into structured input parameters for ice load calculation. Ship owners usually specify the area of operation and cargoes to be transported, and wish to define the most suitable ice class to maximize their profit while maintaining a sufficient level of safety. The designers should then convert such information into operation profiles of the ship, including at least ice condition parameters (thickness, concentration, strength, etc.) and operation parameters (speed, frequency of maneuver, escorted or independent navigation, etc.). These parameters serve as the input for ice load calculation.

2. **Probabilistic modeling of ice loads.** This is the essential part of risk-based design. The idea is to

calculate ice load in a probabilistic manner, so that the long-term extremes can be evaluated in terms of return period, which could then link to different structural limit states. Existing goal- and risk-based design approaches all use the Event-Maximum Method (EMM) proposed by Jordaan et al. (1993) and its variants (Töns et al., 2015; Shamaei et al., 2020; Li et al., 2021). This method applies an exponential distribution to fit the tail of the ice load measurement data and extrapolate to the long term using the Gumbel distribution. This way, long-term extreme loads can be calculated based on the operation profile (i.e., the goal) of the ship.

3. **Modeling of damage and consequence.** Once ice loads have been expressed as a function of return periods, one can then correspond return periods to different limit states of ship structures, including at least Accident Limit State (ALS), Ultimate Limit State (ULS), and Serviceability Limit State (SLS). Different limit states correspond to different severities of consequence. Therefore, proper modeling of consequences is essential to set up criteria for different limit states. Some work has been done through the LRF-funded project CEPOLAR (see Browne et al., 2020; Bergström et al., 2022) on the modeling of economic, ecological, and social consequences. In principle, icegoing ships are allowed to get dented at the hull at an acceptable frequency, since such damage would not affect much of the functionality. The dents can be repaired when the ship is in dry-dock at some cost (see Ehlers et al. 2017). It is therefore important to define the allowable permanent deflection of the hull corresponding to certain return periods.

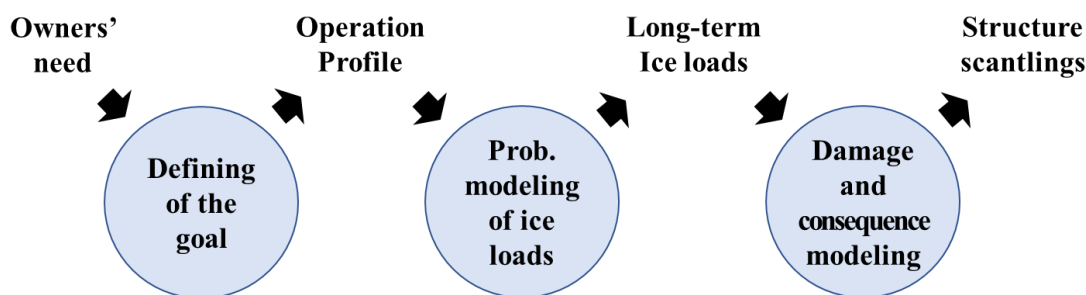


Figure 1. Concept of goal- and risk-based design of polar ships.

Through the above process, one can define the necessary structural scantling to ensure adequate safety

without unnecessary conservatism. The criteria to define limit states (allowable deflection and corresponding

return period) can differ between different structure members (shell, stiffener, girders) and different regions (bow, shoulder, stern).

3. Current gaps towards applicable goal- and risk-based design

Next, we will conduct an analysis of the practicality and knowledge gap of current methods to support goal- and risk-based design.

3.1 Determination of operation profile

It is of primary importance to model the goal properly in a comprehensive and structured way. Ship owners usually provide information about the intended operation area/route and how often the ship operates. This information serves as the initial input to define the goal. The process of modeling the goal can be expressed in the following equation:

$$[\mathbf{v}, \mathbf{M}, \mathbf{t}, \mathbf{IC}] = f(\text{Operation Area, Operation Frequency, Ship performance})$$

where \mathbf{v} denotes ship speed, \mathbf{M} the mode of operation (escorted, independent, etc.), \mathbf{t} the duration of operation, and \mathbf{IC} the ice condition. The bold letters denote that these parameters are vectors rather than single values. Ship performance means the achievable speed at different ice conditions, sometimes characterized by the so-called h - v (ice thickness-ship speed curve). Ship performance is, of course, also a design issue, but for structure design, it serves as the input.

The first thing is to convert the operation area into ice condition parameters, including at least topological parameters (thickness, concentration, floe size, ridging condition, etc.) and mechanical properties (elastic modulus, flexural strength, compressive strength, etc.). These parameters are first of all seasonally varied, in summer/autumn being thin and weak, while in winter/spring being thick and strong. Modern meteorological methods, such as satellite-based imaging techniques, enable long-term recording of ice conditions over large-scale areas. Although providing extensive information about ice distributions, this information is still short in spatial resolution and precision.

An analogy can be made to ocean waves. To date, the sea state of different regions can be quantified in a systematic manner. Wave spectrums have been introduced to model waves in the short term and are able to quantify the stochasticity in wave heights. Long-term statistics of sea states can be summarized via a wave

scatter diagram. This way, long-term wave heights corresponding to return periods can be defined based on the operation profile of the ship. However, in sea ice environment modeling, we are still very far from reaching a similar kind of maturity. The most commonly used way to characterize ice state is still the egg chart proposed by the World Meteorological Organization (WMO, 2014), which is rather coarse with no connection to short-term stochasticity. Long-term variation may be summarized, but a statistical model is still unavailable. Another major issue is that information on statistics of ice mechanical properties, including, e.g., elastic modulus and different strengths, is even rarer and sparser compared to geometrical parameters, with no clue of either short-term or long-term variations. The situation will probably not change in the foreseeable future.

Due to this, a complete and comprehensive characterization of the ice conditions a ship may encounter in its lifetime is practically impossible. The best we can get is an overall view of the average ice conditions with no idea of the extremes. However, in structure design, it is the extremes rather than the averages that drive the design. This is a dilemma that has not been addressed convincingly, even in current polar ship rules. For example, it is stated in the background note of IACS UR (IACS, 2023) that the ice condition parameters used to define the polar classes correspond to a one-year return period. This is really hard to justify, as for example, no studies really prove or indicate that the one-year maximum thickness for a PC1 ship operating in all polar water is 7m.

Therefore, to support goal-based design, the best we can do is maybe to divide the polar water into different regions and define the corresponding average geometrical and mechanical properties of each region. Such an approach has been adopted by Canadian and Russian rules (Transport Canada, 2023; Ministry of Transport of the Russian Federation, 2020), but ice geometrical and mechanical properties are not explicitly given. As systematic measurement of ice mechanical properties over large spatial and temporal scales is impractical, the feasible way is probably to calculate the properties based on hydrological parameters such as sea salinity, temperature, and wind speed. Tarovik et al. (2022) made a recent attempt that divides the polar water along the northeast passage into different regions and calculates the mechanical properties of each region. Naturally, considerable deviations and uncertainties pertain in such methods. To further enhance its applicability, one can calibrate this calculation with

existing (but sparse) measurements to increase its credibility.

To summarize, suppose the ship owner has an idea of where a new ship is to operate and how often, the possible way of defining its goal is to make a table as sketched in Table 2. Here, the input is the operational area, frequency of operation, and overall performance of the ship. The output is then the operational speed,

operational mode, time of operation, as well as ice geometrical and mechanical information. The division of sea areas and seasons can be refined if needed. This table can then serve as the input for long-term ice load modeling. The data in this table do not have to be single values, but can be characterized with distributions reflecting the overall temporal and spatial variation within each category.

Table 2. A possible way of defining the goal (operation profile).

Polar sea area	Season of operation	Frequency of operation	Ice condition	Speed of operation	Mode of operation	Time of operation
	Spring	x_1 transits/year	Thickness, Concentration, Strength, etc.	Based on ship performance curves	Independent / escorted	x hours
Kara Sea	Summer	x_2 transits/year
	Autumn	x_3 transits/year
	Winter	x_4 transits/year

Laptev Sea
...

3.2 Long-term prediction of ice loads

In the design of a ship in waves, theoretical/numerical methods such as strip theory, potential flow, or Computational Fluid Dynamics (CFD) can be applied to calculate motion and structural responses, thereby establishing the Response Amplitude Ratio (RAO) to convert wave spectrum into response spectrum. However, such methodology fails to apply in ice load modeling due to the absence of statistical characterization of ice states. As discussed in Section 3.1, what we have is only an overall description of ice conditions without much information about its extremes. As a result, existing attempts at modeling long-term ice loads have adopted a different approach by using available ice load measurements to quantify their stochasticity. Such methodology is essentially empirical.

The Event-Maximum Method (EMM) has been reckoned as the feasible way to make long-term predictions of ice loads (Ehlers et al, 2017; Bergström et al., 2020). To facilitate long-term ice load modeling, the method divides measured ice loads into segments by defining an event, which can be a ramming process or a fixed time interval, as reviewed by Li et al. (2021). The term maximum means to extract the largest load from each event, which effectively filters the numerous smaller loads that may hamper long-term modeling. An exponential distribution is fitted to the upper tail of the data plotted in an exponential probability paper, and the obtained distribution parameter is linked to the design inputs such as load patch area (Jordaan et al., 1993) and ice conditions (Shamaei et al., 2020). Future extreme loads can then be predicted by extrapolating the parent distribution to the long term via extreme value theory.

This framework is also compatible with other methods, such as the Kujala (1996) method, which uses a Gumbel distribution instead of an exponential distribution for load maxima.

There are still a few gaps before such probabilistic models can effectively support goal- and risk-based design. The first is the consideration of ice conditions. In the original EMM by Jordaan et al. (1993) and its several variants (Taylor et al., 2010; Ralph et al., 2016; Töns et al., 2015), ice condition is not a parameter in the model. This is because ramming is considered as the design scenario and ice loads is not sensitive to ice thickness in ramming (as long as no flexural failure occurs). This makes it not suitable for ships that mainly involve continuous operation. To overcome this, Shamaei et al. (2020) and Li et al. (2021) later extended this method to include ice thickness and concentration, which enables probabilistic modeling of continuous icebreaking. Nonetheless, the mechanical property of ice is still not considered in the results. This is naturally because the models are established upon measurement data, which does not contain along-trip measurements of mechanical properties. However, the drawback needs to be overcome due to the significant influence of mechanical properties on ice loads.

Another gap is the consideration of ship parameters, of which the most important one is the flare angle. Flare angle is a parameter considered by IACS UR, which affects the reduced mass and velocity in glancing impact. It also has a major influence on the flexural failure in continuous icebreaking. Probabilistic models are typically established on one or a few ships, which does not guarantee precision when generalized to other ships.

The third gap is the consideration of ship speed. Although ship speed can be relatively easily measured along the passage, it is usually not effective to include it in the probabilistic model. This is because for a given ship, its speed is determined by the ice condition if we assume the ship mainly operates in full power in ice. Therefore, based on data obtained by one ship, speed is conditional on ice condition, and the variation under the same ice condition is very small, which does not help in establishing a model capable of generalization. The result of Kotilainen et al. (2017) actually shows this issue.

The fourth gap is the consideration of the operation mode. Existing work has not effectively distinguished ice loads under different operation modes, e.g., independent or being escorted. Shamaei et al. (2020) have shown that escorting can have a large impact on the long-term

statistics of ice loads. Consideration of operation mode is an important part of goal-based design and deserves more attention in future work.

The aforementioned drawbacks result in insufficient generalization performance of current probabilistic models, which should be addressed properly. Physics-based model shall play its role in scaling the distribution parameters to other ships by considering ice mechanical properties, ship hull parameters, and differences in performance (e.g., h-v curves). Probabilistic models shall be established separately for independent and escorted operations. These will complete the current probabilistic models to account for the main prevailing parameters. After that, cross-validation using measurements from different ships can provide evidence of the credibility of the developed model. The concept can be illustrated in Figure 2, and can be expressed in the following equation:

$$f_z(z) = P(z; \theta), \text{ where } \theta = \theta_{\text{ref}}(IC_{\text{geo}}, M, t)g(IC_{\text{mec}}, SP, HP)$$

where $P(z|\theta)$ denotes the probability of extreme ice load z (e.g., annual maxima) with distribution parameters θ ; θ is calculated as the product of a reference parameter θ_{ref} and a correction factor g . The reference parameter θ_{ref} is obtained on a reference ship with extensive measurement data, and is a function of the geometrical ice conditions, IC_{geo} and mode of operation, M . The correction factor g accounts for the target ship performance, SP , hull parameters, HP , and the ice mechanical properties of the area it operates in, IC_{mec} .

3.3. Limit state and consequence modeling

Riska and Bridges (2019) discussed the status of applying limit state design to polar ships. The limit states connect the extreme level of ice loads with the extent of damage to stiffened structures. It has been shown by e.g., Körgesaar et al. (2018) that the residual strength after yielding is still abundant to bear higher loads before fracture. According to the background notes of IACS UR (IACS, 2016), the frame damage is calculated based on the three-hinge mechanism and is allowed to happen once per year (as specified in the description of design loads). This contrasts with Finnish-Swedish ice-class rules (TRAFICOM, 2021) where first yielding is selected as the design point with ice loads corresponding to a much shorter return period.

Neither IACS UR nor FSICR indicates what limit state their design points correspond to. This is the main gap towards risk-based design. ISO19906 (ISO, 2010)

recommends the return period of ice action corresponding to different limit states, as listed in Table 3. But for ships and especially different structural members of ships, the applicability of the limit state definition may change due to the mobile feature of ships in comparison to offshore structures. The remaining question is to define what is the permanent deflection of the stiffened panel corresponding to each limit state. With the modern Finite Element Method (FEM), it is not difficult to evaluate the deflection under ice loads. The main difficulty lies in the evaluation of consequences corresponding to the extent of permanent deflection, as consequences to humans, the economy, and the

environment are not straightforward to quantify. Some work has been done on this topic (Browne et al., 2020), but its connection to the limit state is yet to be established. Once this is completed, we can directly connect ice loads of different return periods to the extent of structure damage via limit states, thereby determining the structure scantling. This process can be expressed as:

$$\text{scantlings given limit state} = h(z, d | \text{limit state})$$

where $h(\cdot)$ is the mapping between ice load (z), extent of damage (d), and the required scantlings.

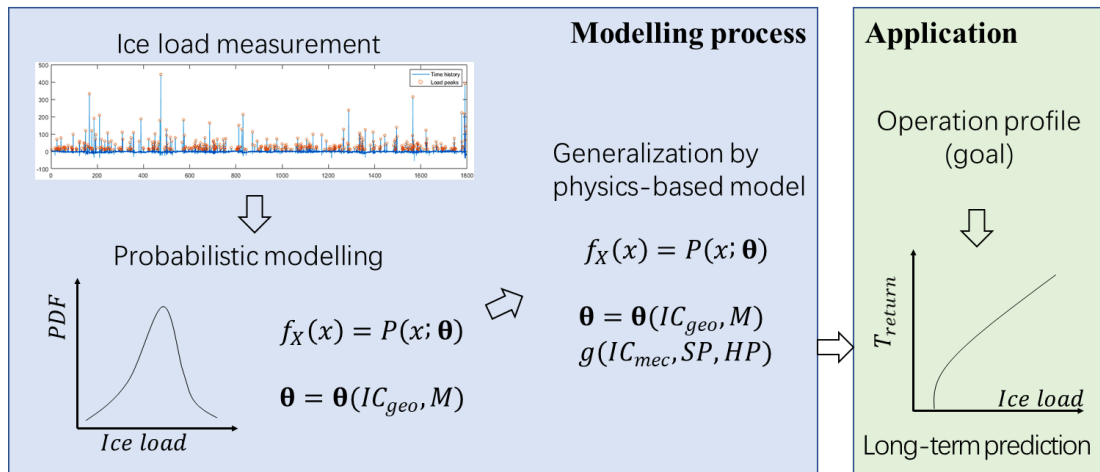


Figure 2. Concept of long-term ice load prediction.

Table 3. Description of limit states as per ISO (2010).

Limit state	Structure damage	Return period
Serviceability limit state	Structure members subject to localized damage	10 years
Ultimate limit state	Some localized inelastic behavior	10^2 years
Accidental limit state	Structural components are allowed to behave plastically	10^3 - 10^5 years

3.4. Further discussion

There are a few more issues to be discussed. The first is whether POLARIS or its goal- and risk-based variants should be implemented as a mandatory regulation. The author tends to believe that it might be difficult to implement a mandatory regulation to select ice class, because this involves mainly the operational level. The owners should be responsible for defining the level of risk and consequence they can bear. POLARIS is goal-

based, but the goal of a ship might change during its operational life if, e.g., market changes. Nonetheless, the mandate may be implemented by local maritime administration authorities, who can define the minimum ice class of ships that can pass through either independently or being escorted to ensure minimal loss to humans and the environment. These considered, the author thinks POLARIS, or its alternatives, is good to stay as a guideline for design.

The current POLARIS is essentially operation-oriented, which is framed to calculate risks rather than to directly guide the user to select an ice class. This is an implicit way of defining risk, with no distinction between probability of failure and associated consequence. The resulting risk indices cannot quantify the level of safety and do not result in the selection of ice class directly. A future approach should be able to help the owners select ice class based on explicit risk assessment.

Ice conditions are becoming more unpredictable and vary yearly. Nonetheless, the overall declining trend is relatively clear over a relatively long period. A goal- and risk-based approach is capable of considering accumulated damage rather than the ultimate bearing capacity, which can effectively average the uncertainties in ice condition prediction in the long term. Considering that ice-induced damages are usually local and associated with mild consequences, goal- and risk-based approaches make it feasible to design the structure from a life-cycle perspective, rather than only considering the most extreme event.

The last issue to discuss here is the realizability of a goal- and risk-based approach in the current conditions and the conditions under which it could be implemented. In the opinion of the author, the first step is to establish a reasonable procedure to define the operation profile, considering both subjective and objective uncertainties. The second is a proper way to generalize probabilistic ice load prediction to include all necessary parameters. The third is a comprehensive method to evaluate risks considering consequences. Besides, benchmarking studies with existing experience of ice navigation are necessary to properly tune the methods to ensure the necessary safety level is maintained. Also, profitability assessment shall be carried out to understand how much economic and environmental benefit can be gained using an alternative design approach.

4. Concluding remarks

This paper makes a short review of the features and existing work on goal- and risk-based design of polar ships and identifies the current gap towards practically applicable design methods. The main gap exists in the establishment of an area-based ice condition table, the generalization of probabilistic ice load methods, and the correspondence of consequence modeling to limit states. The author believes these gaps can be overcome with some effort based on our current knowledge of ice action. Future work is also encouraged in profitability

assessment of using goal- and risk-based approach, considering the reduction in lightweight, profit of extra cargo volume, cost of repair, etc. With these investigations and improvements, goal- and risk-based design may demonstrate its superiority over current prescriptive design approaches and become applicable in ship design practices.

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